# Mathematica 11.3 Integration Test Results

## Test results for the 84 problems in "6.5.2 (e x)^m (a+b sech(c+d x^n))^p.m"

### Problem 18: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x^3}{a+b\,\text{Sech}\left[c+d\,x^2\right]}\,dx$$

Optimal (type 4, 241 leaves, 11 steps):

$$\frac{x^4}{4\,a}-\frac{b\,x^2\,\text{Log}\left[1+\frac{a\,e^{c+d\,x^2}}{b-\sqrt{-a^2+b^2}}\right]}{2\,a\,\sqrt{-a^2+b^2}\,d}+\frac{b\,x^2\,\text{Log}\left[1+\frac{a\,e^{c+d\,x^2}}{b+\sqrt{-a^2+b^2}}\right]}{2\,a\,\sqrt{-a^2+b^2}\,d}-\frac{b\,\text{PolyLog}\left[2,-\frac{a\,e^{c+d\,x^2}}{b-\sqrt{-a^2+b^2}}\right]}{2\,a\,\sqrt{-a^2+b^2}\,d^2}+\frac{b\,\text{PolyLog}\left[2,-\frac{a\,e^{c+d\,x^2}}{b+\sqrt{-a^2+b^2}}\right]}{2\,a\,\sqrt{-a^2+b^2}\,d^2}$$

Result (type 4, 843 leaves):

$$\frac{1}{4\,a\,\left(a+b\,\text{Sech}\left[c+d\,x^2\right]\right)}\left(b+a\,\text{Cosh}\left[c+d\,x^2\right]\right)\left(x^4+\frac{1}{\sqrt{a^2-b^2}\,d^2}\,2\,b\left(2\,\left(c+d\,x^2\right)\,\text{ArcTan}\left[\frac{(a+b)\,\text{Coth}\left[\frac{1}{2}\left(c+d\,x^2\right)\right]}{\sqrt{a^2-b^2}}\right]+\right.\right.$$

$$2\left(c-\text{i}\,\text{ArcCos}\left[-\frac{b}{a}\right]\right)\text{ArcTan}\left[\frac{(a-b)\,\text{Tanh}\left[\frac{1}{2}\left(c+d\,x^2\right)\right]}{\sqrt{a^2-b^2}}\right]+\left(\text{ArcCos}\left[-\frac{b}{a}\right]+\right.$$

$$\left.2\left(\text{ArcTan}\left[\frac{(a+b)\,\text{Coth}\left[\frac{1}{2}\left(c+d\,x^2\right)\right]}{\sqrt{a^2-b^2}}\right]+\text{ArcTan}\left[\frac{(a-b)\,\text{Tanh}\left[\frac{1}{2}\left(c+d\,x^2\right)\right]}{\sqrt{a^2-b^2}}\right]\right)\right)$$

$$\text{Log}\left[\frac{\sqrt{a^2-b^2}\,e^{-\frac{c}{2}-\frac{d\,x^2}{2}}}{\sqrt{2}\,\sqrt{a}\,\sqrt{b+a\,\text{Cosh}\left[c+d\,x^2\right]}}\right]+\left(\text{ArcCos}\left[-\frac{b}{a}\right]-\right.$$

$$\left.2\left(\text{ArcTan}\left[\frac{(a+b)\,\text{Coth}\left[\frac{1}{2}\left(c+d\,x^2\right)\right]}{\sqrt{a^2-b^2}}\right]+\text{ArcTan}\left[\frac{(a-b)\,\text{Tanh}\left[\frac{1}{2}\left(c+d\,x^2\right)\right]}{\sqrt{a^2-b^2}}\right]\right)\right)\text{Log}\left[\right.$$

$$\left.\frac{\sqrt{a^2-b^2}\,e^{\frac{1}{2}\left(c+d\,x^2\right)}}{\sqrt{2}\,\sqrt{a}\,\sqrt{b+a\,\text{Cosh}\left[c+d\,x^2\right]}}\right]-\left(\text{ArcCos}\left[-\frac{b}{a}\right]+2\,\text{ArcTan}\left[\frac{(a-b)\,\text{Tanh}\left[\frac{1}{2}\left(c+d\,x^2\right)\right]}{\sqrt{a^2-b^2}}\right]\right)$$

$$\text{Log}\left[\left((a+b)\left(-a+b+\text{i}\,\sqrt{a^2-b^2}\right)\left(-1+\text{Tanh}\left[\frac{1}{2}\left(c+d\,x^2\right)\right]\right)\right)\right/$$

$$\left.\left(a\left(a+b+\text{i}\,\sqrt{a^2-b^2}\,\text{Tanh}\left[\frac{1}{2}\left(c+d\,x^2\right)\right]\right)\right)\right]-$$

$$\left(\text{ArcCos}\left[-\frac{b}{a}\right]-2\,\text{ArcTan}\left[\frac{(a-b)\,\text{Tanh}\left[\frac{1}{2}\left(c+d\,x^2\right)\right]}{\sqrt{a^2-b^2}}\right]\right)$$

$$\text{Log}\left[\frac{(a+b)\left(a-b+\text{i}\,\sqrt{a^2-b^2}\right)\left(1+\text{Tanh}\left[\frac{1}{2}\left(c+d\,x^2\right)\right]\right)}{a\left(a+b+\text{i}\,\sqrt{a^2-b^2}\,\text{Tanh}\left[\frac{1}{2}\left(c+d\,x^2\right)\right]\right)}\right]+$$

$$\text{i}\left(\text{PolyLog}\left[2,\left(\left(b-\text{i}\,\sqrt{a^2-b^2}\right)\left(a+b-\text{i}\,\sqrt{a^2-b^2}\,\text{Tanh}\left[\frac{1}{2}\left(c+d\,x^2\right)\right]\right)\right)\right/\right.$$

$$\left.\left(a\left(a+b+\text{i}\,\sqrt{a^2-b^2}\,\text{Tanh}\left[\frac{1}{2}\left(c+d\,x^2\right)\right]\right)\right)\right]-$$

$$\text{PolyLog}\left[2,\left(\left(b+\text{i}\,\sqrt{a^2-b^2}\right)\left(a+b-\text{i}\,\sqrt{a^2-b^2}\,\text{Tanh}\left[\frac{1}{2}\left(c+d\,x^2\right)\right]\right)\right)\right/$$

$$\left.\left.\left.\left(a\left(a+b+\text{i}\,\sqrt{a^2-b^2}\,\text{Tanh}\left[\frac{1}{2}\left(c+d\,x^2\right)\right]\right)\right)\right]\right)\right)\text{Sech}\left[c+d\,x^2\right]$$

## Problem 28: Attempted integration timed out after 120 seconds.

$$\int\frac{1}{x\,\left(a+b\,\text{Sech}\left[c+d\,x^2\right]\right)^2}\,dx$$

Optimal (type 8, 21 leaves, 0 steps):

$$\text{Int}\left[\frac{1}{x\,\left(a+b\,\text{Sech}\left[c+d\,x^2\right]\right)^2},\,x\right]$$

Result (type 1, 1 leaves):

???

## Problem 29: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{x^2\,\left(a+b\,\text{Sech}\left[c+d\,x^2\right]\right)^2}\,dx$$

Optimal (type 8, 21 leaves, 0 steps):

$$\text{Int}\left[\frac{1}{x^2\,\left(a+b\,\text{Sech}\left[c+d\,x^2\right]\right)^2},\,x\right]$$

Result (type 1, 1 leaves):

???

## Problem 30: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{x^3\,\left(a+b\,\text{Sech}\left[c+d\,x^2\right]\right)^2}\,dx$$

Optimal (type 8, 21 leaves, 0 steps):

$$\text{Int}\left[\frac{1}{x^3\,\left(a+b\,\text{Sech}\left[c+d\,x^2\right]\right)^2},\,x\right]$$

Result (type 1, 1 leaves):

???

## Problem 50: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{x\,\left(a+b\,\text{Sech}\left[c+d\,\sqrt{x}\right]\right)^2}\,dx$$

Optimal (type 8, 23 leaves, 0 steps):

$$\text{Int}\left[\frac{1}{x\,\left(a+b\,\text{Sech}\left[c+d\,\sqrt{x}\right]\right)^2},\,x\right]$$

Result (type 1, 1 leaves):

???

## Problem 51: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{x^2 \left(a + b\,\text{Sech}\left[c + d\sqrt{x}\,\right]\right)^2}\,dx$$

Optimal (type 8, 23 leaves, 0 steps):

$$\text{Int}\left[\frac{1}{x^2 \left(a + b\,\text{Sech}\left[c + d\sqrt{x}\,\right]\right)^2},\ x\right]$$

Result (type 1, 1 leaves):

???

## Problem 75: Unable to integrate problem.

$$\int (e\,x)^{-1+3\,n} \left(a + b\,\text{Sech}\left[c + d\,x^n\right]\right)\,dx$$

Optimal (type 4, 217 leaves, 11 steps):

$$\frac{a\,(e\,x)^{3\,n}}{3\,e\,n} + \frac{2\,b\,x^{-n}\,(e\,x)^{3\,n}\,\text{ArcTan}\left[e^{c+d\,x^n}\right]}{d\,e\,n} - \frac{2\,i\,b\,x^{-2\,n}\,(e\,x)^{3\,n}\,\text{PolyLog}\left[2,\,-i\,e^{c+d\,x^n}\right]}{d^2\,e\,n} + \frac{2\,i\,b\,x^{-2\,n}\,(e\,x)^{3\,n}\,\text{PolyLog}\left[2,\,i\,e^{c+d\,x^n}\right]}{d^2\,e\,n} + \frac{2\,i\,b\,x^{-3\,n}\,(e\,x)^{3\,n}\,\text{PolyLog}\left[3,\,-i\,e^{c+d\,x^n}\right]}{d^3\,e\,n} - \frac{2\,i\,b\,x^{-3\,n}\,(e\,x)^{3\,n}\,\text{PolyLog}\left[3,\,i\,e^{c+d\,x^n}\right]}{d^3\,e\,n}$$

Result (type 8, 24 leaves):

$$\int (e\,x)^{-1+3\,n} \left(a + b\,\text{Sech}\left[c + d\,x^n\right]\right)\,dx$$

## Problem 78: Unable to integrate problem.

$$\int (e\,x)^{-1+3\,n} \left(a + b\,\text{Sech}\left[c + d\,x^n\right]\right)^2\,dx$$

Optimal (type 4, 363 leaves, 16 steps):

$$\frac{a^2\,(e\,x)^{3\,n}}{3\,e\,n} + \frac{b^2\,x^{-n}\,(e\,x)^{3\,n}}{d\,e\,n} + \frac{4\,a\,b\,x^{-n}\,(e\,x)^{3\,n}\,\text{ArcTan}\left[e^{c+d\,x^n}\right]}{d\,e\,n} - \frac{2\,b^2\,x^{-2\,n}\,(e\,x)^{3\,n}\,\text{Log}\left[1 + e^{2\,(c+d\,x^n)}\right]}{d^2\,e\,n} - \frac{4\,i\,a\,b\,x^{-2\,n}\,(e\,x)^{3\,n}\,\text{PolyLog}\left[2,\,-i\,e^{c+d\,x^n}\right]}{d^2\,e\,n} + \frac{4\,i\,a\,b\,x^{-2\,n}\,(e\,x)^{3\,n}\,\text{PolyLog}\left[2,\,i\,e^{c+d\,x^n}\right]}{d^2\,e\,n} - \frac{b^2\,x^{-3\,n}\,(e\,x)^{3\,n}\,\text{PolyLog}\left[2,\,-e^{2\,(c+d\,x^n)}\right]}{d^3\,e\,n} + \frac{4\,i\,a\,b\,x^{-3\,n}\,(e\,x)^{3\,n}\,\text{PolyLog}\left[3,\,-i\,e^{c+d\,x^n}\right]}{d^3\,e\,n} - \frac{4\,i\,a\,b\,x^{-3\,n}\,(e\,x)^{3\,n}\,\text{PolyLog}\left[3,\,i\,e^{c+d\,x^n}\right]}{d^3\,e\,n} + \frac{b^2\,x^{-n}\,(e\,x)^{3\,n}\,\text{Tanh}\left[c + d\,x^n\right]}{d\,e\,n}$$

Result (type 8, 26 leaves):

$$\int (e\,x)^{-1+3\,n}\,\left(a+b\,\text{Sech}\left[c+d\,x^n\right]\right)^2\,dx$$

## Problem 80: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(e\,x)^{-1+2\,n}}{a+b\,\text{Sech}\left[c+d\,x^n\right]}\,dx$$

Optimal (type 4, 307 leaves, 12 steps):

$$\frac{(e\,x)^{2\,n}}{2\,a\,e\,n}-\frac{b\,x^{-n}\,(e\,x)^{2\,n}\,\text{Log}\left[1+\frac{a\,e^{c+d\,x^n}}{b-\sqrt{-a^2+b^2}}\right]}{a\,\sqrt{-a^2+b^2}\,d\,e\,n}+\frac{b\,x^{-n}\,(e\,x)^{2\,n}\,\text{Log}\left[1+\frac{a\,e^{c+d\,x^n}}{b+\sqrt{-a^2+b^2}}\right]}{a\,\sqrt{-a^2+b^2}\,d\,e\,n}-\frac{b\,x^{-2\,n}\,(e\,x)^{2\,n}\,\text{PolyLog}\left[2,-\frac{a\,e^{c+d\,x^n}}{b-\sqrt{-a^2+b^2}}\right]}{a\,\sqrt{-a^2+b^2}\,d^2\,e\,n}+\frac{b\,x^{-2\,n}\,(e\,x)^{2\,n}\,\text{PolyLog}\left[2,-\frac{a\,e^{c+d\,x^n}}{b+\sqrt{-a^2+b^2}}\right]}{a\,\sqrt{-a^2+b^2}\,d^2\,e\,n}$$

Result (type 4, 859 leaves):

$$
\begin{aligned}
&\frac{1}{2\,a\,e\,n\,\left(a+b\,\text{Sech}\left[c+d\,x^n\right]\right)}\,(e\,x)^{2\,n}\,\left(b+a\,\text{Cosh}\left[c+d\,x^n\right]\right)\\
&\quad\left(1+\frac{1}{\sqrt{a^2-b^2}\;d^2}\,2\,b\,x^{-2\,n}\,\left(2\,\left(c+d\,x^n\right)\,\text{ArcTan}\left[\frac{(a+b)\,\text{Coth}\left[\frac{1}{2}\,\left(c+d\,x^n\right)\right]}{\sqrt{a^2-b^2}}\right]+\right.\right.\\
&\qquad 2\,\left(c-i\,\text{ArcCos}\left[-\frac{b}{a}\right]\right)\,\text{ArcTan}\left[\frac{(a-b)\,\text{Tanh}\left[\frac{1}{2}\,\left(c+d\,x^n\right)\right]}{\sqrt{a^2-b^2}}\right]+\left(\text{ArcCos}\left[-\frac{b}{a}\right]+\right.\\
&\qquad\left.2\,\left(\text{ArcTan}\left[\frac{(a+b)\,\text{Coth}\left[\frac{1}{2}\,\left(c+d\,x^n\right)\right]}{\sqrt{a^2-b^2}}\right]+\text{ArcTan}\left[\frac{(a-b)\,\text{Tanh}\left[\frac{1}{2}\,\left(c+d\,x^n\right)\right]}{\sqrt{a^2-b^2}}\right]\right)\right)\\
&\qquad\text{Log}\left[\frac{\sqrt{a^2-b^2}\;e^{-\frac{c}{2}-\frac{d\,x^n}{2}}}{\sqrt{2}\,\sqrt{a}\,\sqrt{b+a\,\text{Cosh}\left[c+d\,x^n\right]}}\right]+\left(\text{ArcCos}\left[-\frac{b}{a}\right]-\right.\\
&\qquad\left.2\,\left(\text{ArcTan}\left[\frac{(a+b)\,\text{Coth}\left[\frac{1}{2}\,\left(c+d\,x^n\right)\right]}{\sqrt{a^2-b^2}}\right]+\text{ArcTan}\left[\frac{(a-b)\,\text{Tanh}\left[\frac{1}{2}\,\left(c+d\,x^n\right)\right]}{\sqrt{a^2-b^2}}\right]\right)\right)\,\text{Log}\Bigg[\\
&\qquad\frac{\sqrt{a^2-b^2}\;e^{\frac{1}{2}\,\left(c+d\,x^n\right)}}{\sqrt{2}\,\sqrt{a}\,\sqrt{b+a\,\text{Cosh}\left[c+d\,x^n\right]}}\Bigg]-\left(\text{ArcCos}\left[-\frac{b}{a}\right]+2\,\text{ArcTan}\left[\frac{(a-b)\,\text{Tanh}\left[\frac{1}{2}\,\left(c+d\,x^n\right)\right]}{\sqrt{a^2-b^2}}\right]\right)\\
&\qquad\text{Log}\left[\frac{(a+b)\,\left(-a+b+i\,\sqrt{a^2-b^2}\right)\,\left(-1+\text{Tanh}\left[\frac{1}{2}\,\left(c+d\,x^n\right)\right]\right)}{a\,\left(a+b+i\,\sqrt{a^2-b^2}\,\text{Tanh}\left[\frac{1}{2}\,\left(c+d\,x^n\right)\right]\right)}\right]-\\
&\qquad\left(\text{ArcCos}\left[-\frac{b}{a}\right]-2\,\text{ArcTan}\left[\frac{(a-b)\,\text{Tanh}\left[\frac{1}{2}\,\left(c+d\,x^n\right)\right]}{\sqrt{a^2-b^2}}\right]\right)\\
&\qquad\text{Log}\left[\frac{(a+b)\,\left(a-b+i\,\sqrt{a^2-b^2}\right)\,\left(1+\text{Tanh}\left[\frac{1}{2}\,\left(c+d\,x^n\right)\right]\right)}{a\,\left(a+b+i\,\sqrt{a^2-b^2}\,\text{Tanh}\left[\frac{1}{2}\,\left(c+d\,x^n\right)\right]\right)}\right]+\\
&\qquad i\,\left(\text{PolyLog}\left[2,\frac{\left(b-i\,\sqrt{a^2-b^2}\right)\,\left(a+b-i\,\sqrt{a^2-b^2}\,\text{Tanh}\left[\frac{1}{2}\,\left(c+d\,x^n\right)\right]\right)}{a\,\left(a+b+i\,\sqrt{a^2-b^2}\,\text{Tanh}\left[\frac{1}{2}\,\left(c+d\,x^n\right)\right]\right)}\right]-\right.\\
&\qquad\left.\left.\left.\text{PolyLog}\left[2,\frac{\left(b+i\,\sqrt{a^2-b^2}\right)\,\left(a+b-i\,\sqrt{a^2-b^2}\,\text{Tanh}\left[\frac{1}{2}\,\left(c+d\,x^n\right)\right]\right)}{a\,\left(a+b+i\,\sqrt{a^2-b^2}\,\text{Tanh}\left[\frac{1}{2}\,\left(c+d\,x^n\right)\right]\right)}\right]\right)\right)\right)\,\text{Sech}\left[c+d\,x^n\right]
\end{aligned}
$$

## Problem 81: Unable to integrate problem.

$$\int\frac{(e\,x)^{-1+3\,n}}{a+b\,\text{Sech}\left[c+d\,x^n\right]}\,dx$$

Optimal (type 4, 452 leaves, 14 steps):

$$\frac{(e\,x)^{3n}}{3\,a\,e\,n}-\frac{b\,x^{-n}\,(e\,x)^{3n}\,\text{Log}\left[1+\frac{a\,e^{c+d\,x^n}}{b-\sqrt{-a^2+b^2}}\right]}{a\,\sqrt{-a^2+b^2}\,d\,e\,n}+\frac{b\,x^{-n}\,(e\,x)^{3n}\,\text{Log}\left[1+\frac{a\,e^{c+d\,x^n}}{b+\sqrt{-a^2+b^2}}\right]}{a\,\sqrt{-a^2+b^2}\,d\,e\,n}-\frac{2\,b\,x^{-2n}\,(e\,x)^{3n}\,\text{PolyLog}\left[2,-\frac{a\,e^{c+d\,x^n}}{b-\sqrt{-a^2+b^2}}\right]}{a\,\sqrt{-a^2+b^2}\,d^2\,e\,n}+\frac{2\,b\,x^{-2n}\,(e\,x)^{3n}\,\text{PolyLog}\left[2,-\frac{a\,e^{c+d\,x^n}}{b+\sqrt{-a^2+b^2}}\right]}{a\,\sqrt{-a^2+b^2}\,d^2\,e\,n}+\frac{2\,b\,x^{-3n}\,(e\,x)^{3n}\,\text{PolyLog}\left[3,-\frac{a\,e^{c+d\,x^n}}{b-\sqrt{-a^2+b^2}}\right]}{a\,\sqrt{-a^2+b^2}\,d^3\,e\,n}-\frac{2\,b\,x^{-3n}\,(e\,x)^{3n}\,\text{PolyLog}\left[3,-\frac{a\,e^{c+d\,x^n}}{b+\sqrt{-a^2+b^2}}\right]}{a\,\sqrt{-a^2+b^2}\,d^3\,e\,n}$$

Result (type 8, 26 leaves):

$$\int\frac{(e\,x)^{-1+3n}}{a+b\,\text{Sech}[c+d\,x^n]}\,dx$$

## Problem 83: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{(e\,x)^{-1+2n}}{(a+b\,\text{Sech}[c+d\,x^n])^2}\,dx$$

Optimal (type 4, 717 leaves, 23 steps):

$$\frac{(e\,x)^{2n}}{2\,a^2\,e\,n}+\frac{b^3\,x^{-n}\,(e\,x)^{2n}\,\text{Log}\left[1+\frac{a\,e^{c+d\,x^n}}{b-\sqrt{-a^2+b^2}}\right]}{a^2\,(-a^2+b^2)^{3/2}\,d\,e\,n}-\frac{2\,b\,x^{-n}\,(e\,x)^{2n}\,\text{Log}\left[1+\frac{a\,e^{c+d\,x^n}}{b-\sqrt{-a^2+b^2}}\right]}{a^2\,\sqrt{-a^2+b^2}\,d\,e\,n}-\frac{b^3\,x^{-n}\,(e\,x)^{2n}\,\text{Log}\left[1+\frac{a\,e^{c+d\,x^n}}{b+\sqrt{-a^2+b^2}}\right]}{a^2\,(-a^2+b^2)^{3/2}\,d\,e\,n}+\frac{2\,b\,x^{-n}\,(e\,x)^{2n}\,\text{Log}\left[1+\frac{a\,e^{c+d\,x^n}}{b+\sqrt{-a^2+b^2}}\right]}{a^2\,\sqrt{-a^2+b^2}\,d\,e\,n}-\frac{b^2\,x^{-2n}\,(e\,x)^{2n}\,\text{Log}[b+a\,\text{Cosh}[c+d\,x^n]]}{a^2\,(a^2-b^2)\,d^2\,e\,n}+\frac{b^3\,x^{-2n}\,(e\,x)^{2n}\,\text{PolyLog}\left[2,-\frac{a\,e^{c+d\,x^n}}{b-\sqrt{-a^2+b^2}}\right]}{a^2\,(-a^2+b^2)^{3/2}\,d^2\,e\,n}-\frac{2\,b\,x^{-2n}\,(e\,x)^{2n}\,\text{PolyLog}\left[2,-\frac{a\,e^{c+d\,x^n}}{b-\sqrt{-a^2+b^2}}\right]}{a^2\,\sqrt{-a^2+b^2}\,d^2\,e\,n}-\frac{b^3\,x^{-2n}\,(e\,x)^{2n}\,\text{PolyLog}\left[2,-\frac{a\,e^{c+d\,x^n}}{b+\sqrt{-a^2+b^2}}\right]}{a^2\,(-a^2+b^2)^{3/2}\,d^2\,e\,n}+\frac{2\,b\,x^{-2n}\,(e\,x)^{2n}\,\text{PolyLog}\left[2,-\frac{a\,e^{c+d\,x^n}}{b+\sqrt{-a^2+b^2}}\right]}{a^2\,\sqrt{-a^2+b^2}\,d^2\,e\,n}+\frac{b^2\,x^{-n}\,(e\,x)^{2n}\,\text{Sinh}[c+d\,x^n]}{a\,(a^2-b^2)\,d\,e\,n\,(b+a\,\text{Cosh}[c+d\,x^n])}$$

Result (type 4, 2651 leaves):

$$\frac{1}{(a^2-b^2)^{3/2}\,d^2\,n\,(a+b\,\text{Sech}[c+d\,x^n])^2}$$

$$2\,b\,x^{1-2n}\,(e\,x)^{-1+2n}\,(b+a\,\text{Cosh}[c+d\,x^n])^2\left(2\,(i\,c+i\,d\,x^n)\,\text{ArcTanh}\left[\frac{(a+b)\,\text{Cot}\left[\frac{1}{2}\,(i\,c+i\,d\,x^n)\right]}{\sqrt{a^2-b^2}}\right]-\right.$$

$$2\left(i c+\text{ArcCos}\left[-\frac{b}{a}\right]\right)\text{ArcTanh}\left[\frac{(a-b)\,\text{Tan}\left[\frac{1}{2}\left(i c+i d x^n\right)\right]}{\sqrt{a^2-b^2}}\right]+\left(\text{ArcCos}\left[-\frac{b}{a}\right]-\right.$$

$$\left.2 i\left(\text{ArcTanh}\left[\frac{(a+b)\,\text{Cot}\left[\frac{1}{2}\left(i c+i d x^n\right)\right]}{\sqrt{a^2-b^2}}\right]-\text{ArcTanh}\left[\frac{(a-b)\,\text{Tan}\left[\frac{1}{2}\left(i c+i d x^n\right)\right]}{\sqrt{a^2-b^2}}\right]\right)\right)$$

$$\text{Log}\left[\frac{\sqrt{a^2-b^2}\;e^{-\frac{1}{2} i\left(i c+i d x^n\right)}}{\sqrt{2}\;\sqrt{a}\;\sqrt{b+a\,\text{Cosh}\left[c+d x^n\right]}}\right]+\left(\text{ArcCos}\left[-\frac{b}{a}\right]+\right.$$

$$\left.2 i\left(\text{ArcTanh}\left[\frac{(a+b)\,\text{Cot}\left[\frac{1}{2}\left(i c+i d x^n\right)\right]}{\sqrt{a^2-b^2}}\right]-\text{ArcTanh}\left[\frac{(a-b)\,\text{Tan}\left[\frac{1}{2}\left(i c+i d x^n\right)\right]}{\sqrt{a^2-b^2}}\right]\right)\right)$$

$$\text{Log}\left[\frac{\sqrt{a^2-b^2}\;e^{\frac{1}{2} i\left(i c+i d x^n\right)}}{\sqrt{2}\;\sqrt{a}\;\sqrt{b+a\,\text{Cosh}\left[c+d x^n\right]}}\right]-$$

$$\left(\text{ArcCos}\left[-\frac{b}{a}\right]+2 i\,\text{ArcTanh}\left[\frac{(a-b)\,\text{Tan}\left[\frac{1}{2}\left(i c+i d x^n\right)\right]}{\sqrt{a^2-b^2}}\right]\right)$$

$$\text{Log}\left[1-\frac{\left(b-i\sqrt{a^2-b^2}\right)\left(a+b-\sqrt{a^2-b^2}\;\text{Tan}\left[\frac{1}{2}\left(i c+i d x^n\right)\right]\right)}{a\left(a+b+\sqrt{a^2-b^2}\;\text{Tan}\left[\frac{1}{2}\left(i c+i d x^n\right)\right]\right)}\right]+$$

$$\left(-\text{ArcCos}\left[-\frac{b}{a}\right]+2 i\,\text{ArcTanh}\left[\frac{(a-b)\,\text{Tan}\left[\frac{1}{2}\left(i c+i d x^n\right)\right]}{\sqrt{a^2-b^2}}\right]\right)$$

$$\text{Log}\left[1-\frac{\left(b+i\sqrt{a^2-b^2}\right)\left(a+b-\sqrt{a^2-b^2}\;\text{Tan}\left[\frac{1}{2}\left(i c+i d x^n\right)\right]\right)}{a\left(a+b+\sqrt{a^2-b^2}\;\text{Tan}\left[\frac{1}{2}\left(i c+i d x^n\right)\right]\right)}\right]+$$

$$i\left(\text{PolyLog}\left[2,\frac{\left(b-i\sqrt{a^2-b^2}\right)\left(a+b-\sqrt{a^2-b^2}\;\text{Tan}\left[\frac{1}{2}\left(i c+i d x^n\right)\right]\right)}{a\left(a+b+\sqrt{a^2-b^2}\;\text{Tan}\left[\frac{1}{2}\left(i c+i d x^n\right)\right]\right)}\right]-\right.$$

$$\left.\left.\text{PolyLog}\left[2,\frac{\left(b+i\sqrt{a^2-b^2}\right)\left(a+b-\sqrt{a^2-b^2}\;\text{Tan}\left[\frac{1}{2}\left(i c+i d x^n\right)\right]\right)}{a\left(a+b+\sqrt{a^2-b^2}\;\text{Tan}\left[\frac{1}{2}\left(i c+i d x^n\right)\right]\right)}\right]\right)\right)\text{Sech}\left[c+d x^n\right]^2-$$

$$\frac{1}{a^2\left(a^2-b^2\right)^{3/2} d^2 n\left(a+b\,\text{Sech}\left[c+d x^n\right]\right)^2}\,b^3 x^{1-2 n}\,(e x)^{-1+2 n}$$

$$\left(b+a\,\text{Cosh}\left[c+d x^n\right]\right)^2$$

$$\left(2\left(i c+i d x^n\right)\text{ArcTanh}\left[\frac{(a+b)\,\text{Cot}\left[\frac{1}{2}\left(i c+i d x^n\right)\right]}{\sqrt{a^2-b^2}}\right]-\right.$$

$$2\left(i c+\text{ArcCos}\left[-\frac{b}{a}\right]\right)\text{ArcTanh}\left[\frac{(a-b)\,\text{Tan}\left[\frac{1}{2}\left(i c+i d x^n\right)\right]}{\sqrt{a^2-b^2}}\right]+\left(\text{ArcCos}\left[-\frac{b}{a}\right]-\right.$$

$$\left.2 i\left(\text{ArcTanh}\left[\frac{(a+b)\,\text{Cot}\left[\frac{1}{2}\left(i c+i d x^n\right)\right]}{\sqrt{a^2-b^2}}\right]-\text{ArcTanh}\left[\frac{(a-b)\,\text{Tan}\left[\frac{1}{2}\left(i c+i d x^n\right)\right]}{\sqrt{a^2-b^2}}\right]\right)\right)$$

$$\text{Log}\left[\frac{\sqrt{a^2-b^2}\,e^{-\frac{1}{2}i\,(i\,c+i\,d\,x^n)}}{\sqrt{2}\,\sqrt{a}\,\sqrt{b+a\,\text{Cosh}[c+d\,x^n]}}\right]+\left(\text{ArcCos}\left[-\frac{b}{a}\right]+\right.$$

$$\left.2\,i\left(\text{ArcTanh}\left[\frac{(a+b)\,\text{Cot}\left[\frac{1}{2}\,(i\,c+i\,d\,x^n)\right]}{\sqrt{a^2-b^2}}\right]-\text{ArcTanh}\left[\frac{(a-b)\,\text{Tan}\left[\frac{1}{2}\,(i\,c+i\,d\,x^n)\right]}{\sqrt{a^2-b^2}}\right]\right)\right)$$

$$\text{Log}\left[\frac{\sqrt{a^2-b^2}\,e^{\frac{1}{2}i\,(i\,c+i\,d\,x^n)}}{\sqrt{2}\,\sqrt{a}\,\sqrt{b+a\,\text{Cosh}[c+d\,x^n]}}\right]-$$

$$\left(\text{ArcCos}\left[-\frac{b}{a}\right]+2\,i\,\text{ArcTanh}\left[\frac{(a-b)\,\text{Tan}\left[\frac{1}{2}\,(i\,c+i\,d\,x^n)\right]}{\sqrt{a^2-b^2}}\right]\right)$$

$$\text{Log}\left[1-\frac{\left(b-i\,\sqrt{a^2-b^2}\right)\left(a+b-\sqrt{a^2-b^2}\,\text{Tan}\left[\frac{1}{2}\,(i\,c+i\,d\,x^n)\right]\right)}{a\left(a+b+\sqrt{a^2-b^2}\,\text{Tan}\left[\frac{1}{2}\,(i\,c+i\,d\,x^n)\right]\right)}\right]+$$

$$\left(-\text{ArcCos}\left[-\frac{b}{a}\right]+2\,i\,\text{ArcTanh}\left[\frac{(a-b)\,\text{Tan}\left[\frac{1}{2}\,(i\,c+i\,d\,x^n)\right]}{\sqrt{a^2-b^2}}\right]\right)$$

$$\text{Log}\left[1-\frac{\left(b+i\,\sqrt{a^2-b^2}\right)\left(a+b-\sqrt{a^2-b^2}\,\text{Tan}\left[\frac{1}{2}\,(i\,c+i\,d\,x^n)\right]\right)}{a\left(a+b+\sqrt{a^2-b^2}\,\text{Tan}\left[\frac{1}{2}\,(i\,c+i\,d\,x^n)\right]\right)}\right]+$$

$$i\left(\text{PolyLog}\left[2,\frac{\left(b-i\,\sqrt{a^2-b^2}\right)\left(a+b-\sqrt{a^2-b^2}\,\text{Tan}\left[\frac{1}{2}\,(i\,c+i\,d\,x^n)\right]\right)}{a\left(a+b+\sqrt{a^2-b^2}\,\text{Tan}\left[\frac{1}{2}\,(i\,c+i\,d\,x^n)\right]\right)}\right]-\right.$$

$$\left.\left.\left.\text{PolyLog}\left[2,\frac{\left(b+i\,\sqrt{a^2-b^2}\right)\left(a+b-\sqrt{a^2-b^2}\,\text{Tan}\left[\frac{1}{2}\,(i\,c+i\,d\,x^n)\right]\right)}{a\left(a+b+\sqrt{a^2-b^2}\,\text{Tan}\left[\frac{1}{2}\,(i\,c+i\,d\,x^n)\right]\right)}\right]\right)\right)\right)\text{Sech}[c+d\,x^n]^2+$$

$$\left(x^{1-n}\,(e\,x)^{-1+2\,n}\,(b+a\,\text{Cosh}[c+d\,x^n])^2\,\text{Sech}[c]\,\text{Sech}[c+d\,x^n]^2\right.$$

$$\left.\left(a^2\,d\,x^n\,\text{Cosh}[c]-b^2\,d\,x^n\,\text{Cosh}[c]+2\,b^2\,\text{Sinh}[c]\right)\right)\Big/$$

$$\left(2\,a^2\,(a-b)\,(a+b)\,d\,n\,(a+b\,\text{Sech}[c+d\,x^n])^2\right)-$$

$$\left(b^2\,x^{1-2\,n}\,(e\,x)^{-1+2\,n}\,(b+a\,\text{Cosh}[c+d\,x^n])^2\,\text{Sech}[c]\,\text{Sech}[c+d\,x^n]^2\right.$$

$$\left(a\,\text{Cosh}[c]\,\text{Log}\left[b+a\,\text{Cosh}[c]\,\text{Cosh}[d\,x^n]+a\,\text{Sinh}[c]\,\text{Sinh}[d\,x^n]\right]-\right.$$

$$\left.\left.a\,d\,x^n\,\text{Sinh}[c]+\frac{2\,a\,b\,\text{ArcTan}\left[\frac{a\,\text{Sinh}[c]+(-b+a\,\text{Cosh}[c])\,\text{Tanh}\left[\frac{d\,x^n}{2}\right]}{\sqrt{-b^2+a^2\,\text{Cosh}[c]^2-a^2\,\text{Sinh}[c]^2}}\right]\text{Sinh}[c]}{\sqrt{-b^2+a^2\,\text{Cosh}[c]^2-a^2\,\text{Sinh}[c]^2}}\right)\right)\Big/$$

$$\left(a\left(a^2-b^2\right)d^2\,n\left(a+b\,\mathrm{Sech}\left[c+d\,x^n\right]\right)^2\left(a^2\,\mathrm{Cosh}[c]^2-a^2\,\mathrm{Sinh}[c]^2\right)\right)+$$
$$\left(b^2\,x^{1-n}\,(e\,x)^{-1+2\,n}\left(b+a\,\mathrm{Cosh}\left[c+d\,x^n\right]\right)\mathrm{Sech}[c]\,\mathrm{Sech}\left[c+d\,x^n\right]^2\left(b\,\mathrm{Sinh}[c]-a\,\mathrm{Sinh}\left[d\,x^n\right]\right)\right)\Big/$$
$$\left(a^2\left(-a+b\right)\left(a+b\right)d\,n\left(a+b\,\mathrm{Sech}\left[c+d\,x^n\right]\right)^2\right)+$$
$$\frac{b^2\,x^{1-n}\,(e\,x)^{-1+2\,n}\left(b+a\,\mathrm{Cosh}[c+d\,x^n]\right)^2\mathrm{Sech}[c+d\,x^n]^2\,\mathrm{Tanh}[c]}{a^2\left(-a^2+b^2\right)d\,n\left(a+b\,\mathrm{Sech}[c+d\,x^n]\right)^2}-$$
$$\left(2\,b^3\,x^{1-2\,n}\,(e\,x)^{-1+2\,n}\,\mathrm{ArcTan}\left[\frac{\left(-a+b\right)\mathrm{Tanh}\left[\frac{1}{2}\left(c+d\,x^n\right)\right]}{\sqrt{a^2-b^2}}\right]\right.$$
$$\left.\left(b+a\,\mathrm{Cosh}\left[c+d\,x^n\right]\right)^2\mathrm{Sech}\left[c+d\,x^n\right]^2\,\mathrm{Tanh}[c]\right)\Big/$$
$$\left(a^2\left(a^2-b^2\right)^{3/2}d^2\,n\left(a+b\,\mathrm{Sech}\left[c+d\,x^n\right]\right)^2\right)$$

## Problem 84: Attempted integration timed out after 120 seconds.

$$\int\frac{(e\,x)^{-1+3\,n}}{\left(a+b\,\mathrm{Sech}[c+d\,x^n]\right)^2}\,dx$$

Optimal (type 4, 1284 leaves, 32 steps):

$$\frac{(e\,x)^{3\,n}}{3\,a^2\,e\,n}+\frac{b^2\,x^{-n}\,(e\,x)^{3\,n}}{a^2\,\left(a^2-b^2\right)\,d\,e\,n}-\frac{2\,b^2\,x^{-2\,n}\,(e\,x)^{3\,n}\,\text{Log}\left[1+\frac{a\,e^{c+d\,x^n}}{b-\sqrt{-a^2+b^2}}\right]}{a^2\,\left(a^2-b^2\right)\,d^2\,e\,n}+$$

$$\frac{b^3\,x^{-n}\,(e\,x)^{3\,n}\,\text{Log}\left[1+\frac{a\,e^{c+d\,x^n}}{b-\sqrt{-a^2+b^2}}\right]}{a^2\,\left(-a^2+b^2\right)^{3/2}\,d\,e\,n}-\frac{2\,b\,x^{-n}\,(e\,x)^{3\,n}\,\text{Log}\left[1+\frac{a\,e^{c+d\,x^n}}{b-\sqrt{-a^2+b^2}}\right]}{a^2\,\sqrt{-a^2+b^2}\,d\,e\,n}-$$

$$\frac{2\,b^2\,x^{-2\,n}\,(e\,x)^{3\,n}\,\text{Log}\left[1+\frac{a\,e^{c+d\,x^n}}{b+\sqrt{-a^2+b^2}}\right]}{a^2\,\left(a^2-b^2\right)\,d^2\,e\,n}-\frac{b^3\,x^{-n}\,(e\,x)^{3\,n}\,\text{Log}\left[1+\frac{a\,e^{c+d\,x^n}}{b+\sqrt{-a^2+b^2}}\right]}{a^2\,\left(-a^2+b^2\right)^{3/2}\,d\,e\,n}+$$

$$\frac{2\,b\,x^{-n}\,(e\,x)^{3\,n}\,\text{Log}\left[1+\frac{a\,e^{c+d\,x^n}}{b+\sqrt{-a^2+b^2}}\right]}{a^2\,\sqrt{-a^2+b^2}\,d\,e\,n}-\frac{2\,b^2\,x^{-3\,n}\,(e\,x)^{3\,n}\,\text{PolyLog}\left[2,-\frac{a\,e^{c+d\,x^n}}{b-\sqrt{-a^2+b^2}}\right]}{a^2\,\left(a^2-b^2\right)\,d^3\,e\,n}+$$

$$\frac{2\,b^3\,x^{-2\,n}\,(e\,x)^{3\,n}\,\text{PolyLog}\left[2,-\frac{a\,e^{c+d\,x^n}}{b-\sqrt{-a^2+b^2}}\right]}{a^2\,\left(-a^2+b^2\right)^{3/2}\,d^2\,e\,n}-\frac{4\,b\,x^{-2\,n}\,(e\,x)^{3\,n}\,\text{PolyLog}\left[2,-\frac{a\,e^{c+d\,x^n}}{b-\sqrt{-a^2+b^2}}\right]}{a^2\,\sqrt{-a^2+b^2}\,d^2\,e\,n}-$$

$$\frac{2\,b^2\,x^{-3\,n}\,(e\,x)^{3\,n}\,\text{PolyLog}\left[2,-\frac{a\,e^{c+d\,x^n}}{b+\sqrt{-a^2+b^2}}\right]}{a^2\,\left(a^2-b^2\right)\,d^3\,e\,n}-\frac{2\,b^3\,x^{-2\,n}\,(e\,x)^{3\,n}\,\text{PolyLog}\left[2,-\frac{a\,e^{c+d\,x^n}}{b+\sqrt{-a^2+b^2}}\right]}{a^2\,\left(-a^2+b^2\right)^{3/2}\,d^2\,e\,n}+$$

$$\frac{4\,b\,x^{-2\,n}\,(e\,x)^{3\,n}\,\text{PolyLog}\left[2,-\frac{a\,e^{c+d\,x^n}}{b+\sqrt{-a^2+b^2}}\right]}{a^2\,\sqrt{-a^2+b^2}\,d^2\,e\,n}-\frac{2\,b^3\,x^{-3\,n}\,(e\,x)^{3\,n}\,\text{PolyLog}\left[3,-\frac{a\,e^{c+d\,x^n}}{b-\sqrt{-a^2+b^2}}\right]}{a^2\,\left(-a^2+b^2\right)^{3/2}\,d^3\,e\,n}+$$

$$\frac{4\,b\,x^{-3\,n}\,(e\,x)^{3\,n}\,\text{PolyLog}\left[3,-\frac{a\,e^{c+d\,x^n}}{b-\sqrt{-a^2+b^2}}\right]}{a^2\,\sqrt{-a^2+b^2}\,d^3\,e\,n}+\frac{2\,b^3\,x^{-3\,n}\,(e\,x)^{3\,n}\,\text{PolyLog}\left[3,-\frac{a\,e^{c+d\,x^n}}{b+\sqrt{-a^2+b^2}}\right]}{a^2\,\left(-a^2+b^2\right)^{3/2}\,d^3\,e\,n}-$$

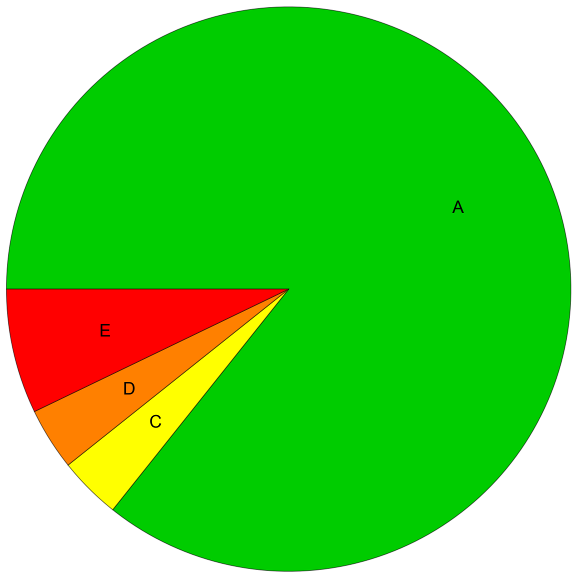
$$\frac{4\,b\,x^{-3\,n}\,(e\,x)^{3\,n}\,\text{PolyLog}\left[3,-\frac{a\,e^{c+d\,x^n}}{b+\sqrt{-a^2+b^2}}\right]}{a^2\,\sqrt{-a^2+b^2}\,d^3\,e\,n}+\frac{b^2\,x^{-n}\,(e\,x)^{3\,n}\,\text{Sinh}\left[c+d\,x^n\right]}{a\,\left(a^2-b^2\right)\,d\,e\,n\,\left(b+a\,\text{Cosh}\left[c+d\,x^n\right]\right)}$$

Result (type 1, 1 leaves):

???

# Summary of Integration Test Results

84 integration problems




A - 72 optimal antiderivatives

B - 0 more than twice size of optimal antiderivatives

C - 3 unnecessarily complex antiderivatives

D - 3 unable to integrate problems

E - 6 integration timeouts